<u>Progetto di ricerca</u> correlato all'assegno

The new fellow will work on topics central to the recently funded PRIN 2022 project:

"Low-rank Structures and Numerical Methods in Matrix and Tensor Computations and their Application" proposal code 20227PCCKZ, CUP J53D23003620006.

More precisely, the project focuses on the following material.

Numerical (multi-)linear algebra is central to many computational methods for complex networks, stochastic processes, machine learning, and numerical solution of PDEs. The matrices (or tensors) encountered in applications are often rank-structured: approximately low-rank, or with low-rank blocks, or low-rank modifications of `simpler' matrices. Identifying and exploiting rank structure is crucial for achieving optimal performance. Below are some of the main areas targeted by our proposal.

LOW RANK APPROXIMATION AND UPDATES:

Given a matrix A, constructing a low-rank approximation means finding two tall and skinny matrices U, V such that ||A - UV^T|| is small. If the norm is the spectral or Frobenius norm, an optimal solution can be computed using a truncated singular value decomposition (SVD). This approach requires O(n^3) floating point operations for an n x n matrix A, which is prohibitive for large-scale problems. Approximate versions of the SVD have been designed to overcome this drawback. These include variants of the Lanczos method and adaptive cross approximation. Recently, randomized methods have gained traction, being more robust than the aforementioned approaches. They only require a few matrix-vector products with A and A^T, and to accept a failure probability that can be made arbitrarily small by slightly increasing the cost of the method. The analogous problem for tensors (arrays with more than 2 indices) is much harder: the set of rank k tensors is not closed, making the minimization problem ill-defined. No explicit solution is available for the best approximant (when it exists), as no direct analogue of the SVD is known. For this reason, several concepts of rank and related compression strategies have been proposed over the years.

If a computational problem can be reduced to an easier subproblem by a low-rank correction, often we can solve the simple problem first, and then reconstruct only the difference with the actual solution in the form of a low-rank update. This is the case for linear systems (Sherman-Morrison formula), QR factorizations, and, recently, matrix functions and matrix equations. The latter two tasks need fundamentally different techniques and accurate spectral information. To overcome these limitations we envision the derivation of randomized low-rank approximation techniques. An advance in this area can be used for updating centrality measures of networks after the addition or removal of nodes or edges.

MATRIX AND TENSOR EQUATIONS IN PDEs:

Array structures, in the form of matrix and tensor equations, naturally stem from the computational treatment of many application problems, nonetheless in the past they have often gone unnoticed or, not sufficiently exploited in the PDE community. Matrix-oriented discretizations of two or three dimensional differential equations were already discussed as early as in 1960 as an intuitive representation of the continuous equation setting, and abandoned in favor of their vector form, at that time computationally more feasible. However, it is now acknowledged that solving the matrix equation can have great numerical advantages -- computational costs and memory requirements -- while it allows to preserve important solution properties such as symmetry and definiteness, even at low accuracies. We aim to adopt this fundamental principle to go from two to three dimensions and up in the matrix and tensor treatment of PDEs with separable and non-separable coefficients, with representative discrete domains. This will allow us to completely bypass the vector formulation of the linear problem, so as to avoid memory allocations that exponentially grow with the tensor dimensions of the data. Explicit tensor computations - even with only three-modes - will lead to similar breakthroughs represented by matrix computations and matrix equation solving for many application problems. The proposed tensor solvers will allow the solution, also in the case of an approximation, to preserve structural properties of the problem, the way direct matrix solvers are now able to do for two-dimensional problems.

DATA SCIENCE TOPICS:

Low-rank structure plays a crucial role also in many other areas. Network science problems, such as clustering and community detection and the construction of preconditioners for solving network-related linear systems, can also be tackled using low-rank approximation techniques, but little work has been done so far in this direction.

The Fellow will be involved in all topics above, and will work under the supervision of members of the UniBO work team, i.e., Valeria Simoncini, Nicola Mastronardi, Davide Palitta, Michele Ruggeri.

<u>Composizione</u> dei <u>membri</u> della <u>commissione</u> dell'eventuale Bando

La valutazione comparativa dei candidati sarà effettuata da una Commissione giudicatrice formata da: - Prof.ssa / Prof. Valeria Simoncini (Presidente)

Prof.ssa / Prof. Davide Palitta (segretario verbalizzante)

Prof.ssa / Prof. Nicola Mastronardi, IAC-CNR (componente)

Prof.ssa / Prof . Beatrice Meini, Universita' di Pisa (PI locale dell'unita' della Universita' di Pisa. (eventuale membro supplente).

<u>Requisiti di ammissione</u>

Alle selezioni sono ammessi a partecipare i candidati, anche cittadini di Paesi non appartenenti alla Unione Europea, in possesso di adeguato curriculum scientifico professionale e di:

- Dottorato di ricerca in Matematica, Matematica Applicata, Scientific Computing, Computer Science, o titolo equivalente corredato da un'adeguata produzione scientifica conseguito in Italia o all'estero;

- è previsto un colloquio;

- in caso di colloquio indicare la modalità: online;

- non è prevista una valutazione di competenza della lingua inglese;